charge density distribution changes and tends to the profile for an infinitely long tube, shown by curve 2 in Fig. 4. This curve is given by the first term on the right-hand side of expression (2.4) for q.

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ELECTRIFICATION OF A METAL BODY IN AN AEROSOL FLOW WITH A SOLID DISPERSE PHASE IN THE PRESENCE OF A CORONA DISCHARGE FROM THE BODY

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UDC 532.5:537

The electrification of a metal body in a flow of uncharged monodisperse aerosol with a solid disperse phase is investigated within the framework of continuum mechanics [1]. The corona discharge from the body is taken into account. We consider cases of well-conducting aerosol particles, for which the electric charge relaxation time is much greater than the time of impact with the body. A closed system of equations and boundary conditions describing the electrification of the body is obtained. We determine the main dimensionless parameters affecting the electrification of the body. We obtain expressions for the electrification current, the maximum corona current, the floating charge and potential of the body, the maximum corona overvoltage, and the characteristic time for establishment of the floating charge on the body. The main dimensionless characteristics of electrification of a sphere with a spark gap are calculated.

1. We consider a metal body with a spark gap in a steady flow of uncharged monodisperse aerosol with a solid disperse phase. As is known [2], the aerosol particles are charged by collisions with the body. The body consequently acquires an electric charge that is opposite in sign to the particle charge. This effect is observed when bodies move through clouds, precipitation, and a dust-laden atmosphere [3]. It can be used in electric probes designed for measuring the parameters of aerosol flows [4].

Using the methods of continuum mechanics [1, 5] we will consider the averaged motion of a monodisperse aerosol flow past a body as the interpenetrating motion of two continuous media – gas and aerosol particles. We assume that the concentration of the latter is fairly low and their effect on the gas motion can be neglected. Then, in the investigation of the electrification of bodies the motion of the gas can be regarded as prescribed. The averaged motion of the aerosol particles before collision with the body is described by the following equations:

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$$m\left(v^k\partial/\partial x^k\right)\mathbf{v} = 6\pi\mu a\left(1+\frac{1}{6}\mathrm{Re}^{2/3}|\mathbf{u}-\mathbf{v}|^{2/3}\right)(\mathbf{u}-\mathbf{v}),$$
$$\partial\left(\eta v^k\right)/\partial x^k = 0,\ \mathbf{v}|_{x^3=-\infty} = u^0\mathbf{e}_3,\ \eta|_{x^3=-\infty} = \eta^0, \tag{1.1}$$

where m, a, η, and $\mathbf{v}$ are the mass, radius, concentration, and velocity of the aerosol particles; $\mathrm{Re} = 2a\rho u^0/\mu$, Reynolds number of the aerosol particle; μ, ρ, and $\mathbf{u}$, viscosity, density, and velocity of the gas; x^k (k = 1, 2, 3), a Cartesian coordinate system in which the x^3 axis has the same direction as the aerosol flow undisturbed by the body; $\mathbf{e}_3$, a unit vector parallel to the x^3 axis; u^0, velocity of the undisturbed aerosol flow; η^0, concentration of aerosol particles in it. It follows from relations (1.1) that

$$\mathbf{v} = u^0\mathbf{v}^*(x^{*k},\ \mathrm{St},\ \mathrm{Re}),\ \eta = \eta^0\eta^*(x^{*k},\ \mathrm{St},\ \mathrm{Re}); \tag{1.2}$$
$$\mathrm{St}(v^{*k}\partial/\partial x^{*k})\mathbf{v}^* = (1+(1/6)\mathrm{Re}^{2/3}|\mathbf{u}^*-\mathbf{v}^*|^{2/3})(\mathbf{u}^*-\mathbf{v}^*), \tag{1.3}$$
$$\partial\left(\eta^* v^{*k}\right)/\partial x^{*k} = 0,\ \mathbf{v}^*|_{x^{*3}=-\infty} = \mathbf{e}_3,\ \eta^*|_{x^{*3}=-\infty} = 1,$$
$$\mathrm{St} = mu^0/(6\pi\mu aR),\ \mathbf{u}^* = \mathbf{u}/u^0,\ x^{*k} = x^k/R,$$

where 2R is the characteristic dimension of the body; St is the dimensionless Stokes number. The dimensionless functions $\mathbf{v}^*(x^{*k})$, $\eta^*(x^{*k})$ for a given distribution of dimensionless gas velocity $\mathbf{u}^*(x^{*k})$ are completely determined by the Stokes and Reynolds numbers.

The increase in electric charge of the aerosol particle due to collision with the body (Δe_p) depends on the physical properties of the materials of the body and particle, the size of the particle, the velocity immediately before impact, and the electric field $\mathbf{E}$ at the impact point on the body surface. Collisions of aerosol particles with the body lead to the flow of an electric current on it;

$$J_+ = \int_S \eta v_\nu \Delta e_p(v_\nu,\ E_\nu)\,ds,\ E_\nu = (\mathbf{E}\nu),\ v_\nu = (\mathbf{v}\nu). \tag{1.4}$$

Here ν is the external normal to the body surface; the integration is taken over the part of the body surface S from which the aerosol particles are reflected; repeat collisions of the particles with the body are ignored (calculations show that their contribution to the electric current J_+ is small).

We denote by E' the characteristic value of the electric field, which has an appreciable effect on Δe_p, and by Φ^0 the striking potential of the corona discharge from the body in the absence of external fields. Since the potential of the initially uncharged body is opposite in sign to the particle charge increment Δe_p, it is obvious that $\Phi^0\Delta e_p < 0$. Henceforth, we confine ourselves to the case where the external field is absent and the conditions

$$|\varepsilon^{-1}\Delta e_p^0\eta^0 R| \ll |\Phi^0|/R \ll E' \tag{1.5}$$

are fulfilled, where ε is the dielectric constant of the aerosol; Δe_p^0 is the characteristic change in charge of the aerosol particle on impact with the body. The inequalities (1.5) mean that in the considered case the electric field has no effect at all on Δe_p or the electrification current, and the electric field produced by reflected aerosol particles has no effect on the corona discharge. Relations (1.5) are always fulfilled for fairly rarefield aerosols (e.g., atmospheric aerosols) and spark gaps with low striking potential.

It follows from the physical sense of Δe_p that it is defined only at points on the part S of the body surface. However, we can formally introduce the function $\Delta e_p(x^k)$ at any point in space through which a streamline of the aerosol particles impinging on the body passes. For this we assume that the function $\Delta e_p(x^k)$ is constant along the streamlines of the aerosol particles and is equal to the change in electric charge of the particles moving through them when the particles collide with the body. For streamlines of aerosol particles that do not collide with body we put $\Delta e_p = 0$. On the basis of the second relation in (1.1) the function $\Delta e_p(x^k)$ defined in this way satisfies the equality

$$\partial(\Delta e_p\eta v^k)/\partial x^k = 0. \tag{1.6}$$

We consider the set of portions of the aerosol particle streamlines terminating on the body. They form a stream tube that is bounded by a lateral surface S' and cuts out a part S on the body surface. In the impinging flow (where $x^3 \to -\infty$) we draw a plane Π perpendicular to its velocity, from which this stream tube cuts out a region S_0. This region is called the capture cross section. We integrate relation (1.6) over the volume bounded by the surface $S + S' + S_0$, using the Ostrogradskii–Gauss formula and the equality (1.4). As a result we obtain a formula connecting the electrification current J_+ with the values of the parameters in the incident aerosol flow:

$$J_+ = -\eta^0 u^0 \int\limits_{S_0} \Delta e_p ds = -\eta^0 u^0 \int\limits_{S_0} \Delta e_p (v_\nu (b_1, b_2))\, db_1 db_2, \tag{1.7}$$

where b_1 and b_2 are direction parameters defining the aerosol particle streamlines. As b_1, b_2 we take the Cartesian coordinates x^1, x^2 in plane Π.

The equation of electrification of the initially uncharged body has the form

$$dQ/dt = J_+ - J_-,\ Q = \varepsilon C \Phi,\ Q|_{t=0} = 0, \tag{1.8}$$

where t is the time; Q, Φ, C, charge, potential, and capacitance of the body; J_-, electric current of the corona discharge. In sufficiently rarefied aerosols, where the first inequality of (1.5) is fulfilled, we have the following empirical ratio for J_-,†

$$J_- = \alpha(C_1 \varepsilon b \Phi / l + C_2 \varepsilon u^0)(\Phi - \Phi^0),$$
$$\alpha = \begin{cases} 0, & \Phi^0 \leqslant \Phi \leqslant 0,\ \Delta e_p \gtrless 0, \\ 1, & \Phi \leqslant \Phi^0 \leqslant 0,\ \Delta e_p \gtrless 0, \end{cases} \tag{1.9}$$

in which b is the mobility of the corona discharge ions ($b\Phi^0 > 0$); l is the characteristic length of the spark gap; C_1, C_2 are dimensionless coefficients which depend on the geometry of the body.

2. Introducing the dimensionless quantities $\Phi^* = \Phi/\Phi^0$, $J^*_\pm = J_\pm / \varepsilon u^0 \Phi^0$, $\Delta e^*_p = \Delta e_p / \Delta e^0_p$, $C^* = C/R$, $t^* = u^0 t/R$, $b^*_1 = b_1/R$, $b^*_2 = b_2/R$, we write relations (1.7)-(1.9) in the form

$$C^* d\Phi^*/dt^* = BJ^* - J^*_-,\ J^* = \frac{1}{S_M} \int\limits_{S_0} \Delta e^*_p db_1 db_2,$$
$$J^*_- = \alpha \left(\frac{C_1}{\mathrm{Pe}_E} \Phi^* + C_2 \right)(\Phi^* - 1),\ \mathrm{Pe}_E = \frac{l u^0}{b\Phi^0} > 0,\ B = -\frac{\eta^0 \Delta e^0_p S_M}{\varepsilon \Phi^0} > 0, \tag{2.1}$$

where S_M is the area of the central section of the body; Pe_E is the electric Péclet number; B is a dimensionless parameter characterizing the ratio of the electric field of the aerosol to the electric field of the body. On the basis of the first inequality in (1.5) we have $|B| \ll 1$, and since $\Delta e^0_p \Phi^0 < 0$, then $B > 0$.

The presence of the small parameter B in Eq. (2.1) means that we can directly write expressions for the maximum overvoltage of the corona discharge and the maximum spark gap current, attained when $d\Phi^*/dt^* = 0$;

$$\max(\Phi/\Phi^0 - 1) = B(C_1/\mathrm{Pe}_E + C_2)^{-1} J^*,$$
$$\max J_- = J_+ = \varepsilon u^0 \Phi^0 B J^*. \tag{2.2}$$

The values of the floating potential Φ' and charge Q' of the body and the characteristic time τ' for their establishment are also found from (2.1):

$$\Phi' = \Phi^0 [1 + BJ^* (C_1/\mathrm{Pe}_E + C_2)^{-1}],\ Q' = \varepsilon C \Phi',\ \tau' = C/(u^0 B J^*).$$

Thus, all the main characteristics of electrification of the body are expressed in terms of the function J^*, given by the second equality of (2.1), which depends on the change in charge of the aerosol particles on collision with the body $\Delta e_p(v_\nu)$. Various expressions can be proposed for Δe_p [4, 6, 7]. We confine ourselves henceforth to the case of well-conducting aerosol particles, where the relaxation time of the electric charge in the particle $\tau_e = \varepsilon_p/(4\pi\sigma_p)$ is much less than the time of impact τ of the particle with the body, and the case of poorly conducting aerosol particles, when $\tau_e = \varepsilon_p/(4\pi\sigma_p) \gg \tau$ (here ε_p and σ_p are the dielectric constant and conductivity of the aerosol particle).

3. Ignoring the effect of the electric field, we determine the change in charge of well-conducting, initially uncharged particles from the formula [6]

$$\Delta e_p = C_\varphi \varphi_c (1 - e^{-A}),\ A = \sigma_p \tau S_c / (C_\varphi d),$$
$$S_\varphi = a(0.577 + 0{,}5 \ln(2a/d)),\ \Delta e^0_p \equiv C_\varphi \varphi_c, \tag{3.1}$$

†V. V. Ushakov, Dissertation for the Degree of Doctor of Technical Sciences: Electrogasdynamic Streams and Electrostatic Charge Control Systems for Aircraft [in Russian], KIIGA, Kiev (1978).

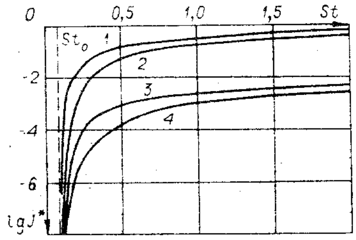

Fig. 1

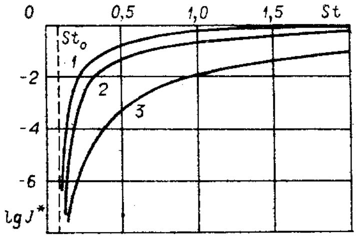

Fig. 2

where φ_c is the contact potential difference for the particles and body materials; S_c is the mean area of contact of the particle with the body during the collision; d is a constant with the dimension of length, whose value is of the same order as the Debye radius of the particle material. The collision parameters τ, S_c contained in relation (3.1) can be found on the basis of the quasistatic theory of impact of an aerosol particle with a body, which in this case can be regarded as a half-space. For a normal elastic collision we have

$$\tau = 3h/|v_\nu|,\ S_c = 2ah,\ h = a\left[\frac{5}{4}\pi\rho_p v_\nu^2\left((1-\nu_p^2)/E_p^Y + (1-\nu_b^2)/E_b^Y\right)\right]^{2/5}, \tag{3.2}$$

where ρ_p is the density of the aerosol particle; ν_p, E_p^Y (ν_b, E_b^Y) are the Poisson's ratio and Young's modulus of the particle (body).

In view of the relations (3.1), (3.2), (1.2), and (1.3), the expression (2.1) for J^* can be put in the form

$$J^*(\mathrm{St}, \mathrm{Re}, A_0) = \frac{1}{S_M^*}\int_{S_0^*}\left[1 - \exp\left(-A_0|v_\nu^*(\mathrm{St}, \mathrm{Re}, b_1^*, b_2^*)|^{3/5}\right)\right]db_1^* db_2^*,$$
$$S_M^* = S_M/R^2,\ S_0^* = S_0/R^2,\ b_{1,2}^* = b_{1,2}/R,\ A_0 = A|_{v_\nu = u_0}. \tag{3.3}$$

The function $v_\nu^*(\mathrm{St}, \mathrm{Re}, b_1^*, b_2^*)$ contained in relation (3.3) was determined by computer calculation of the velocity field of the aerosol particles and their streamlines on the basis of the first equation of (1.3). We then calculated the integral on the right-hand side of relation (3.3). Figure 1 shows a plot of log J^* against the Stokes number St for well-conducting particles in the case of potential flow past a sphere of radius R [1) Re = 10, $A_0 = 5$; 2) Re = 100, $A_0 = 5$; 3) Re = 10, $A_0 = 0.01$; 4) Re = 100, $A_0 = 0.01$]. When St = ∞ it follows from the first equation of (1.3) that $v^* =$ const and for $|v_\nu^*|$ we obtain the expression $|v_\nu^*| = (1 - b_1^{*2} - b_2^{*2})^{1/2}$, which is valid for any conditions of gas flow past a sphere. In this case the value of J^* is found analytically

$$J^* = 2\int_0^1\left[1 - \exp\left(-A_0\left(\sqrt{1-b^{*2}}\right)^{3/5}\right)\right]b^* db^* = 1 - \frac{10}{3}A_0^{-10/3}\gamma\left(\frac{10}{3}, A_0\right).$$

Here γ is an incomplete gamma function: $b^* = (b_1^{*2} + b_2^{*2})^{1/2}$.

4. Ignoring the effect of the electric field, we determine the change in charge of poorly conducting ($\tau_e \gg \tau$), initially uncharged particles from the formula [7]

$$\Delta e_p = -2\pi^{-1/2}C(\pi/2)\,en_0\Sigma\sqrt{D\tau},\ \Delta e_p^0 \equiv \Delta e_p|_{v_\nu = u_0}, \tag{4.1}$$

where $C(\pi/2) = 0.78$ is the value of the Fresnel integral; $\Sigma = \pi a h$ is the maximum area of contact of the particle with the body; e, D, n_0 are the charge, diffusion coefficient, and volume concentration of charge carriers in the particle involved in the reaction on the surface of the body when a particle collides with it. Relation (4.1) is valid for an infinite rate of this reaction, when the charged-particle concentration on the surface of contact of the particle with the body becomes zero. For a normal elastic collision the quantities τ and h contained in relation (4.1) are determined from formulas (3.2).

In the considered case of poorly conducting particles the expression for J^*, in view of the equalities (4.1), (3.2), (1.2), and (1.3), has the form

$$J^*(\mathrm{St}, \mathrm{Re}) = \frac{1}{S_M^*}\int_{S_0^*}|v_\nu^*(\mathrm{St}, \mathrm{Re}, b_1^*, b_2^*)|^{7/10}db_1^* db_2^*. \tag{4.2}$$

Figure 2 shows a computer-calculated plot of log J* against the Stokes number St for poorly conducting particles in the case of potential flow past a sphere of radius R. Curves 1-3 correspond to $\mathrm{Re} = 10$, 10^2, and 10^3. When $\mathrm{St} = \infty$ we have

$$J^* = 2\int_0^1 (\sqrt{1-b^{*2}})^{7/10}\, b^* db^* = \frac{20}{27}.$$

Figures 1 and 2 indicate that there is a critical value of the Stokes number ($\mathrm{St}_0 > 0$), at which J* becomes zero. When the Stokes number is less than St_0 the particles do not reach the body surface. In this case $J^* \equiv 0$ and there is no electrification of the body.

As an example we consider the electrification of a spherical body of diameter $2R = 10$ m in an aerosol flow of ice particles with diameter $a = 10^{-4}$ m, concentration $\eta^0 = 10^8\ \mathrm{m}^{-3}$, and flow velocity $u^0 = 100$ m/sec. For pure ice $\varepsilon_p = 72$, $\sigma_p = 4 \cdot 10^{-7}\ \Omega^{-1} \cdot \mathrm{m}^{-1}$, $e = 1.6 \cdot 10^{-19}$C, $n_0 \simeq 10^{19}$–$10^{20}\ \mathrm{m}^{-3}$, $E_p^Y = 3 \cdot 10^9\ \mathrm{N/m^2}$, $\nu_p = 0.3$. In this case the inequality $\tau \lessdot 10^{-6}$ sec $\ll \tau_e = 1.6 \cdot 10^{-3}$ sec is fulfilled and the theory expounded in Paragraph 4 is applicable. For these numerical values of the parameters we have $\mathrm{St} = 2$, $\mathrm{Re} = 10^3$, $J^* = 10^{-1}$, $\Delta e_p^0 = -5 \cdot 10^{-16}$ C, $J/S_M = 5 \cdot 10^{-7}\ \mathrm{A/m^2}$. Such current densities are actually observed when bodies move in clouds and precipitation [2].

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APPLICATION OF THE MULTIPLE-SCALE METHOD IN THE PROBLEM OF WAVES ON THE SURFACE OF A LIQUID

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Sretenskii [1] has used the method of integral transforms to solve the problem of waves on the surface of a viscous incompressible liquid of inifinite depth. In the low-viscosity case Potetyunko and Strubshchik [2] have constructed asymptotic expansions that are valid in finite time intervals.

In this article we consider the planar Cauchy–Poisson problem for the linearized Navier–Stokes equations in application to the motion of an incompressible low-viscosity liquid under the action of an initial elevation of the free surface:

$$\begin{gathered}
\partial \mathbf{v}/\partial t = -\nabla p + \varepsilon^2 \Delta \mathbf{v},\ \operatorname{div} \mathbf{v} = 0,\\
p = p_r + \lambda z,\ \mathbf{v} = 0,\ \zeta = \zeta_*(x)\ (t = 0),\ -p + \lambda\zeta + 2\varepsilon^2 \partial v_z/\partial z = 0\ \ (z = 0),\\
\partial\zeta/\partial t = v_z,\ \partial v_x/\partial z + \partial v_z/\partial x = 0\ \ (z = 0),\\
(\mathbf{v},\ \partial\mathbf{v}/\partial x,\ p,\ \partial p/\partial x,\ \zeta_*) \to 0,\ |x| \to \infty,\\
\mathbf{v} = 0 \qquad (z = -H).
\end{gathered} \tag{1}$$

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